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COMMENT

Eden trees on the Sierpinski gasket

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Abstract. We have studied Eden trees on the Sierpinski gasket and obtained its fractal and spectral dimensions to be $\ln[(7-\sqrt{5})/2]/\ln 2$ and 1 respectively.

Recently, studies of different statistical processes on fractal lattices have been given a lot of attention. Self-avoiding walks (SAW) on percolation clusters (Chakrabarti and Kertész 1981), the Ising model on SAW (Chakrabarti and Bhattacharya 1983) and diffusion-limited aggregations (DLA) on percolation clusters (Meakin 1984) are examples of studies on random fractals, whereas for deterministic fractals studies of SAW on the Sierpinski gasket (Klein and Seitz 1984, Rammal *et al* 1984), the dilute Ising model on Sierpinski carpets (Boccara and Havlin 1984) and phase transitions on finitely and infinitely ramified fractals (Gefen *et al* 1984a, b) are only a few examples.

A wide variety of different cluster growth models by non-equilibrium processes have been proposed and studied in recent times (Family and Landau 1984). Among these models the Eden process (Eden 1961) is the simplest. Here a cluster grows by adding particles one after another to the perimeter or the growing cluster with equal probability, resulting in compact aggregates. Recently Dhar and Ramaswamy (1985) have studied diffusion on Eden trees. During the growth of these trees, those sites on the perimeter which have more than one particle in the nearest neighbours are excluded. The resulting cluster has a loopless structure. Though these structures are also compact, it is still interesting to study these clusters since the diffusion process on them is characterised by a non-trivial spectral dimension. Performing a numerical study by a generalised node-counting method and by simulation of random walks on these trees, they observed the spectral dimension d_s of these trees to be ~ 1.22 in two dimensions (Dhar and Ramaswamy 1985).

We have studied these Eden trees on the Sierpinski gasket. We find the fractal dimension d_f and spectral dimension d_s of these clusters to be $\ln[(7-\sqrt{5})/2]/\ln 2$ and 1 respectively, whereas the fractal substrate (the Sierpinski gasket) has the corresponding values $\ln 3/\ln 2$ and $2\ln 3/\ln 5$ (Rammal and Toulouse 1983).

The restriction imposed on the Eden process to get trees, i.e. no site on the cluster perimeter can have more than one particle in the neighbourhood, is a severe restriction for trees on Siepinski gaskets. These trees cannot have any branches and will be like linear chains. More specifically, Eden trees on the Sierpinski gasket will be neighbour-avoiding walks (NAW), which is a self-avoiding random walk in which the walker cannot come to the neighbouring site of the previous path (see figure 1). On Euclidean lattices NAW behave in the same way as SAW (Gaunt *et al* 1980). We have studied the

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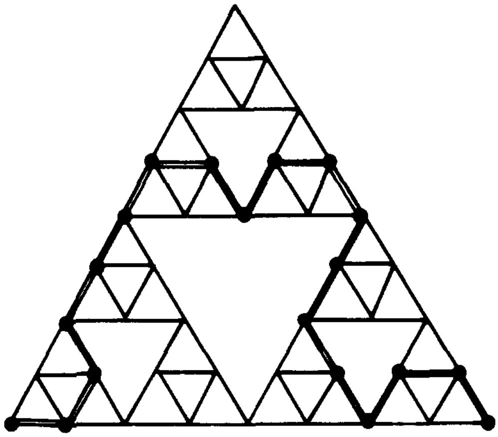

Figure 1. Eden tree on the Sierpinski gasket. Black dots on the gasket sites constitute the cluster. Nearest-neighbour cluster sites are connected by double lines to obtain the walk picture.

statistics of NAW on the Sierpinski gasket by the method of Rammal *et al* (1984) in the following way.

The unit cell ABC of the gasket of scale factor $b=2$ is shown in figure 2. From this figure we see that only two NAW will succeed in reaching the point B, starting from the vertex A and not going via the third vertex C. We associate a weight g with each step of the walk in this cell and the corresponding renormalised weight in the next higher cell will be denoted by g'. The recursion relation connecting g' and g is

$$g' = g^2 + g^3. \tag{1}$$

The non-trivial fixed point of this transformation is

$$g^* = (\sqrt{5}-1)/2. \tag{2}$$

Therefore the connectivity constant $\mu(=1/g^*)$ is 1.618 . . ., and the correlation length index ν will be

$$\begin{aligned} \nu &= \ln b/\ln(\partial g'/\partial g)|_{g=g^*} \\ &= \ln 2/\ln(2g^* + 3g^{*2}) \\ &= \ln 2/\ln[(7-\sqrt{5})/2]. \end{aligned} \tag{3}$$

Therefore the fractal dimension d_f of NAW or Eden trees on the Sierpinski gasket which is the reciprocal of ν is equal to 1.2521 . . . . This value is the same for ordinary SAW on this gasket.

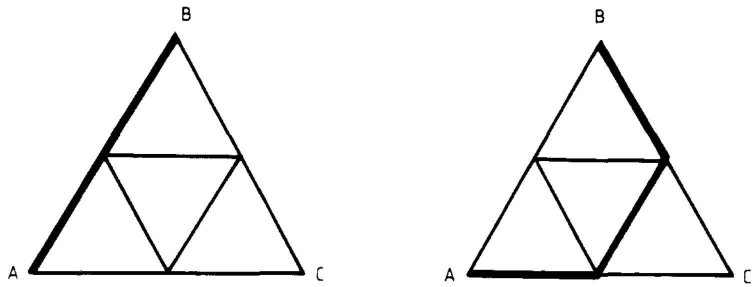

Figure 2. The only two possible neighbour-avoiding walks from site A to site B.

To get a value for the spectral dimension of these trees we imagine that the particles are of equal mass and they are connected by springs with equal force constants. The density of excitation frequencies of this system has the functional form

$$\rho(\omega) \sim \omega^{d_s - 1}. \tag{4}$$

In our case the NAW chain is just equivalent to a compact one-dimensional linear chain and therefore d_s will be equal to its Euclidean dimension, i.e. equal to 1.

This work would be of relevance for studying Eden trees on percolation clusters.

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